Linear Relationships Teacher Guide and Keys

Teach the meanings before choosing a response

Target: construct and interpret a linear relationship from stated conditions and distinct observations; relate change rate, initial value and representations; compare outputs at common inputs and keep domain and linearity assumptions explicit. Selected Grade 8 fit assumes prior labeled rates and proportional-relationship reasoning. Do not call this a complete Grade 8 course, all-function unit or validated assessment.

A two-point calculation does not establish that a process is linear. When the task supplies a linear model, use that condition to determine its unique line. When only observations are supplied, distinguish a conditional linear prediction from an unconditional future claim. The meaning of b as the contextual output at zero requires zero to belong to the allowed domain.

Practical teaching sequence

- Use Starting task without discussion if entry is uncertain. Use earlier ratio/per-one teaching for direction and units, and earlier proportional work for quantity/coordinate/model meanings.
- Rate from changes: model paired differences over unequal intervals. Say “output change per input change” and use the same subtraction order. Show a decrease as a signed subtraction and a constant output as zero change.
- Build a rule and graph: recover b from observed y minus m times observed x. Verify both points, interpret units and plot only the allowed continuous segment. Introduce y = mx + b rather than assuming earlier symbolic fluency.
- Compare rate and output: compute rates and initial values separately, evaluate both at a common input, then find equality. The supplied worked comparison models the simple equation steps; formal systems-of-equations coverage is outside scope.

Allow time to practice and discuss each meaning before the independent check. Arithmetic errors can be addressed separately from a sound model; if fraction or signed arithmetic is not ready, teach it locally and return with different observations. No practical experiment is required.

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Criteria, response teaching and exposure

Evidence sought	What counts; what needs a different response
Change rate	Uses matched signed output/input differences from distinct inputs, with requested units. A y/x quotient including a start is different reasoning.
Initial value and rule	Recovers b, verifies both given points and explains its quantity units. Distinguishes a nonzero observed total from the zero-input value.
Representations and assumptions	Rule, table and plotted/described points refer to the same quantities and domain. Notes when zero or a later input is not allowed, or when linearity is only assumed.
Comparison	Separates rate, starting value and current output; compares a common input and explains an equality and its domain. Distinguishes signed rate from magnitude of a decrease.

Do not infer a stable misconception from one wrong answer. Ask a neutral question such as “What changed between those two readings?” or “Which amount belongs to time zero?” If rate is unclear, use Match the input changes. If b is unclear, use Recover the starting value. If the output comparison is unsupported, use Compare at the same input. Each file supplies modeling and new learner practice. Secure work moves to consolidation or transfer rather than obligatory reteaching.

Read-aloud, larger print, a calculator, extra time or an oral/table/point-description response may preserve this reasoning target. Record support. Supplying differences, the rule, key points or equality reasoning changes independence. If graph drawing itself is a local target, a verbal description provides different evidence; state that change. Both graph figures have equivalent quantity/point descriptions.

Keep unused starting/check/follow-up/transfer pages and all keys with the teacher. After substantive help or answer exposure, use that task as practice. Choose a different unused task after another learning opportunity. These checks are not equated forms and do not establish mastery, diagnosis or instructional efficacy.

Linear Relationships Teacher Guide and Keys

Keys Starting task and first rate practice

Starting task

1: 8 ÷ 4 = 2 liters per minute; time is horizontal and water vertical. This supplied empty-start constant-rate model supports the familiar proportional interpretation. 2: m = (23 − 14)/(5 − 2) = 9/3 = 3 liters per minute; b = 14 − 3 × 2 = 8 liters. V = 3t + 8, with 14 and 23 at the original inputs. 14/2 = 7 includes the starting 8 liters. 3: B has the greater rate, 3 versus 2 liters per minute. At t = 1, A = 11 and B = 5 liters, so A has more total then. The starting amounts affect that comparison.

Rate from changes

Model: (9 − 5)/(2 − 0) = 2 and (15 − 9)/(5 − 2) = 2 liters per minute; the different input intervals account for different output jumps. 1 guided: (13 − 4)/3 = 3 and (25 − 13)/4 = 3 meters per minute. 2 practice: (11.5 − 8.5)/(3 − 1) = 3/2 and (16 − 11.5)/(6 − 3) = 4.5/3 = 1.5 liters per minute. The quotient 8.5/1 includes the start and is not the rate; the matching rule is V = 1.5t + 7.

3 decrease: (8 − 14)/(3 − 1) = −6/2 = −3 liters per minute; the remaining amount drops by 3 liters over one more minute under the model, reaching 5 liters at t = 4. The implied starting amount is 17 liters. 4 constant: (9 − 9)/(4 − 1) = 0 liters per minute. The amount stays 9 liters; V = 9 is linear with m = 0 and b = 9, not a zero-output model.

Accept exact equivalent fraction/decimal values, labeled difference diagrams or an oral explanation with the same quantitative meanings. Reversing both subtraction orders gives the same rate. Reversing only one gives a sign error.

Linear Relationships Teacher Guide and Keys

Keys Build a rule and graph

Worked model: m = 1.5 liters per minute and b = 7 liters; V = 1.5t + 7 verifies (2,10) and (4,13). 1 guided: m = (13 − 7)/(4 − 1) = 2 liters per minute; b = 7 − 2 × 1 = 5 liters; V = 2t + 5. Both original points fit.

2 representation: m = (18 − 8)/(6 − 2) = 10/4 = 2.5 = 5/2 liters per minute; b = 8 − 2.5 × 2 = 3 liters. V = 2.5t + 3. Completed table at t = 0,2,4,6,8: 3,8,13,18,23 liters. Time is horizontal and total water vertical. Plot or accurately describe those points and their straight continuous segment on 0 ≤ t ≤ 8. The vertical grid step is 2 liters; odd and fractional heights are intentional. (0,3) gives the modeled start. This total is not proportional because its start is nonzero; rate and y/x differ.

3 decrease: m = (8 − 13)/(4 − 2) = −5/2 = −2.5 liters per minute. b = 13 − (−2.5 × 2) = 18 liters; V = 18 − 2.5t. It gives 13 and 8 at t = 2 and 4. At t = 5 it gives 5.5 liters, within the domain. The negative sign belongs to the change rate, not the positive initial amount.

4 restricted domain: m = (17 − 11)/(5 − 2) = 2 liters per minute and b = 11 − 2 × 2 = 7 liters. Expression V = 2t + 7 matches both points. Its algebraic value at zero is 7, but zero is outside 2 ≤ t ≤ 8. It does not establish an actual zero-time amount under those supplied conditions; a continued model through zero would be an extra assumption.

A correct rule with missing requested units or graph evidence is partly sound evidence. Note the missing component rather than rejecting the entire model. Do not silently extrapolate a restricted graph to a physical initial state.

Linear Relationships Teacher Guide and Keys

Keys Signed reference and zero start

Build a rule and graph, signed model: [1 − (−3)]/(4 − 2) = 2 degrees Celsius per minute. b = −3 − 2 × 2 = −7 degrees Celsius. C = 2t − 7 returns −3 and 1 at t = 2 and 4. Increasing temperature and a below-zero start are compatible.

5 signed-reference practice: m = [7 − (−2)]/(4 − 1) = 9/3 = 3 degrees Celsius per minute. b = −2 − 3 × 1 = −5 degrees Celsius; C = 3t − 5. At t = 1 the output is −2; at t = 4 it is 7. Positive m means warming in the constructed model; negative b means below zero degrees Celsius at the admitted zero-time reference. The negative start does not mean a negative rate.

6 zero start: m = (15 − 6)/(5 − 2) = 9/3 = 3 liters per minute; b = 6 − 3 × 2 = 0 liters. V = 3t verifies both points. It is proportional on this domain because all totals follow one multiplier and zero gives zero. The constant 9-liter model has rate 0 but start 9: its graph is horizontal and its nonzero constant output is not proportional to elapsed time. The special rule V = 0 has both m and b zero and satisfies V = 0t; a zero rate alone does not settle proportionality.

Follow-up check 4: m = [2 − (−4)]/(4 − 1) = 6/3 = 2 degrees Celsius per minute; b = −4 − 2 × 1 = −6 degrees Celsius. C = 2t − 6 verifies −4 at minute 1 and 2 at minute 4. At minute 3 it gives 0 degrees Celsius. Positive m describes warming; negative b describes the below-zero temperature at the allowed zero-time reference.

Treat the temperature units as Celsius values at a specified reference, not water amounts. An initial value can be negative, zero or positive when the quantity permits those values. Do not impose a negative physical water amount on the earlier water models.

Linear Relationships Teacher Guide and Keys

Keys Compare rate and output

1 guided: A’s rate is (18 − 8)/(6 − 2) = 2.5 liters per minute; b = 8 − 2.5 × 2 = 3 liters. A = 2.5t + 3. B = t + 9, with rate 1 and start 9. A has the greater rate. At t = 0, B is 9 versus A 3; at t = 2, B is 11 versus A 8; at t = 6, A is 18 versus B 15. Equality at t = 4 gives 13 liters each. B is greater before 4 and A after 4, within 0 ≤ t ≤ 6. The graph, rule and points agree.

2 practice: D’s rate is (10 − 8)/(3 − 1) = 1 and (13 − 10)/(6 − 3) = 1 liter per minute; b = 8 − 1 = 7 liters. C = 2t + 2 and D = t + 7. Equality requires 2t + 2 = t + 7, giving t = 5 minutes and 12 liters. D is greater before 5, C after 5, within 0 ≤ t ≤ 8. C’s greater rate does not make its total greater at every time.

4 equal rates: E and F each change by 2 liters per minute. E − F = (2t + 5) − (2t + 1) = 4 liters at every allowed time, so they never have equal totals on 0 to 6. Equal rates preserve the starting difference.

3 two-record question: both possibilities give 6 at minute 1 and 12 at minute 3. At minute 5 the full line 3t + 3 gives 18 liters, while the changed process t + 9 gives 14. Under an explicit full linear assumption, the two distinct inputs give m = (12 − 6)/(3 − 1) = 3 and b = 6 − 3 = 3, selecting the line. The finite record alone does not establish that global assumption. Accept a conditional 18-liter prediction with its assumption, or an explanation that the unconditioned future is undetermined.

Equivalent equation steps, inspection with exact verification, a labeled table or graph-based equality are acceptable. A graph estimate needs enough precision for the requested equality; exact expressions can verify it. Do not treat a comparison outside the given domain as a supplied physical reading.

Linear Relationships Teacher Guide and Keys

Keys Targeted response practice

Match the input changes

Model: both intervals give 2 output units per input unit despite jumps 4 and 6. Practice: (24 − 8)/(5 − 1) = 16/4 = 4 and (36 − 24)/(8 − 5) = 12/3 = 4 meters per minute. Larger output jumps accompany longer intervals. 24/5 = 4.8 includes the starting amount and is not the change rate. A matching rule is L = 4t + 4; this verification is useful but not required if the rate response is complete.

Recover the starting value

Model: m = 2, b = 7, V = 2t + 7 verifies both given observations. Practice: b = 16 − (−1.5 × 2) = 19 liters. V = 19 − 1.5t gives 16 liters at t = 2 and 11.5 liters at t = 5. Negative m describes decreasing remaining water; it does not make b negative. Zero is included in this domain, so 19 is its modeled starting amount.

Compare at the same input

Model: A rate 4 is greater than B rate 2, but at minute 1 B has 9 versus A 5 liters. Equality t = 3 gives 13 each. Practice: C rate 1, start 12; D rate 3, start 2. At minute 1, C = 13 and D = 5; at minute 6, C = 18 and D = 20. Equality t + 12 = 3t + 2 gives 10 = 2t, so t = 5 and each has 17 liters. C is greater before 5 and D after 5 on the stated 0-to-6 domain. The claim “C always has more” fails after 5.

After this practice, choose a different unused next task under the same target. If a calculation was coached or a rule supplied, record that help and do not present the task as unaided evidence.

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Linear Relationships Teacher Guide and Keys

Keys Independent check

1: m = (19.5 − 10.5)/(6 − 2) = 9/4 = 2.25 liters per minute. b = 10.5 − 2.25 × 2 = 6 liters. V = 2.25t + 6, or V = (9/4)t + 6. The rule gives 10.5 and 19.5 at the original inputs. Completed table at t = 0,2,6,8: 6,10.5,19.5,24 liters. Graph/point description has time horizontal and liters vertical, with a straight continuous segment over 0 to 8. The 2-liter grid step requires half- and quarter-step positions; retain exact amounts. This total is not proportional to time: zero gives 6, and y/x is not the same at nonzero inputs.

2: W has the greater rate, 3 versus 2.25 liters per minute. At t = 1, V = 8.25 and W = 4.5 liters, so V is greater. At t = 8, V = 24 and W = 25.5, so W is greater. Equality: 2.25t + 6 = 3t + 1.5 gives 4.5 = 0.75t, hence t = 6 and each total 19.5 liters. V is greater before 6 and W after 6, within the stated domain. A greater rate does not mean greater output at every input.

3: The two observations alone do not establish the minute-8 value. The 24-liter prediction follows if the same linear relationship is explicitly assumed through minute 8. Many non-linear or changing processes can fit both observations. A conditional prediction is defensible; an unqualified physical certainty from the two points alone is not.

Record rate, b, interpretation, representations, comparison and assumptions separately. Do not infer diagnosis or full-topic mastery from this set. Accept exact equivalent numbers and agreed response modes; missing a requested representation is different from an incorrect model.

Linear Relationships Teacher Guide and Keys

Keys Follow-up check

1: m = (9.5 − 16.5)/(6 − 2) = −7/4 = −1.75 liters per minute. b = 16.5 − (−1.75 × 2) = 20 liters. V = 20 − 1.75t verifies both observations. Completed table at t = 0,2,6,8: 20,16.5,9.5,6 liters. Graph or exact point description has a decreasing straight segment on 0 to 8. The rate means a decrease of 1.75 liters each minute; it does not mean negative starting water. The nonzero start rules out proportionality of this remaining total to elapsed time.

2: W’s signed rate −0.75 is greater than V’s −1.75. V loses more per minute in magnitude: 1.75 versus 0.75 liters. At t = 1, V = 18.25 and W = 16.25 liters, so V is greater. At t = 8, V = 6 and W = 11, so W is greater. Equality: 20 − 1.75t = 17 − 0.75t gives 3 = t; at t = 3 both have 14.75 liters. V is greater before 3, W after 3, within the domain. More starting water does not establish greater output throughout it.

3: Minute 10 is outside the stated domain. Algebraically extending V gives 20 − 1.75 × 10 = 2.5 liters, but the supplied model does not establish an actual reading there. That value is a conditional prediction if the same linear model is extended through minute 10. Accept an explanation that the actual outside-domain amount is unspecified, with or without the conditional calculation.

The additional signed-reference task is keyed on the Signed reference and zero start key page.

This set introduces decreasing models and signed-rate comparison; it is not an equal-difficulty or psychometrically equated version of the independent check. Compare the actual criteria and support, not just totals of correct answers.

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Keys Later transfer and use

1 panels: m = (11 − 6)/(6 − 2) = 5/4 = 1.25 meters per complete panel; b = 6 − 1.25 × 2 = 3.5 meters. L = 1.25n + 3.5 verifies 6 and 11 at n = 2 and 6. Four panels give 8.5 meters. The nonzero reserve means total is not proportional to complete-panel count. n = 2.5 is not an allowed whole-panel count; the algebraic value 6.625 meters would belong to a changed interpretation, not the supplied complete-panel model.

2 temperatures: m = (14 − 20)/(5 − 2) = −6/3 = −2 degrees Celsius per minute. Expression C = 24 − 2t matches both points. Its algebraic zero-time value is 24 degrees Celsius, but zero lies outside 2 ≤ t ≤ 8, so that expression alone does not establish an actual temperature at zero. D = 13 has rate 0 degrees Celsius per minute, not temperature zero. At t = 2, C = 20 > D = 13; at t = 8, C = 8 < D = 13. Equality 24 − 2t = 13 gives 2t = 11, so t = 5.5 minutes, within the continuous domain, and both are 13 degrees Celsius.

Accept exact fraction/decimal equivalents, labeled calculations, diagrams or oral reasoning that preserve the requested model meanings and comparison. A fractional time is allowed in the continuous-temperature model; a fractional complete-panel count is not allowed in the first model.

Sources and reuse

All models, numerical data, graphs and learner statements are original Mentor Teaching instructional constructions, not empirical observations. Selected Grade 8 fit references rate/initial-value modeling and comparison across representations in CCSS 8.F.B.4 and 8.F.A.2: https://www.thecorestandards.org/Math/Content/8/F/. This is a scope reference, not complete-standard or course alignment. Print and adapt under the site Terms of Use, retaining attribution and identifying changes. Word edits can change pagination; check print preview.

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Fictional teaching response — sound for the named task

Named task: Independent check 1: supplied linear model through (2, 10.5) and (6, 19.5)

Fictional response — sound for the named task: “The rate is (19.5 − 10.5) ÷ (6 − 2) = 2.25 L/min. At zero there are 10.5 − 2.25 × 2 = 6 L, so V = 2.25t + 6. It gives 10.5 at 2 minutes and 19.5 at 6 minutes. The table is 6, 10.5, 19.5, 24 liters at 0, 2, 6, 8 minutes. Total water is not proportional to time because its start is 6 L.”

Visible evidence: A signed change rate, recovered starting value, checked rule, complete table and justified proportionality conclusion within the supplied 0-to-8-minute linear model.

Not established: That two isolated observations alone establish linear behavior, or the comparisons requested separately in problem 2.

Clarify: Ask why using the supplied linear-model condition is necessary for the prediction at 8 minutes.

Next teaching/evidence: Use the separate Follow up check after another learning opportunity; reserve Later transfer.

This response is invented for teacher interpretation. Accept defensible alternatives. Record access, prior exposure and the actual response; do not assign an ability label or claim mastery from this example.

Follow up check

Reserve: Later transfer

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Fictional teaching response — partially developed

Named task: Independent check 1: supplied linear model through (2, 10.5) and (6, 19.5)

Fictional response — partially developed: “The rate is 2.25 liters per minute.”

Visible evidence: The correct rate value and units.

Not established: Its calculation, starting amount, rule, point checks, table or proportionality explanation.

Clarify: Ask for the two changes used, then what quantity remains after subtracting two minutes of increase from 10.5 liters.

Next teaching/evidence: If starting-value recovery is unclear, use Recover the starting value before giving the follow-up check you have kept aside.

This response is invented for teacher interpretation. Accept defensible alternatives. Record access, prior exposure and the actual response; do not assign an ability label or claim mastery from this example.

Recover the starting value

Reserve: Later transfer

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Fictional teaching response — plausible error

Named task: Independent check 1: supplied linear model through (2, 10.5) and (6, 19.5)

Fictional response — plausible error: “The rate is 19.5 ÷ 6 = 3.25 L/min, so V = 3.25t.”

Visible evidence: An accurately calculated total/time quotient at one displayed point.

Not established: The change rate or a rule fitting both points; the proposed rule gives 6.5 instead of 10.5 liters at t = 2.

Clarify: Ask the learner to test the proposed rule at both points and name the input change that matches the output change.

Next teaching/evidence: Use Match the input changes: divide 24 − 8 by 5 − 1 and 36 − 24 by 8 − 5 in the supplied cord practice. Both are 4 meters per minute; contrast these with total/input division. Reserve the follow-up and later tasks.

This response is invented for teacher interpretation. Accept defensible alternatives. Record access, prior exposure and the actual response; do not assign an ability label or claim mastery from this example.

Match the input changes

Reserve: Later transfer