Proportional Relationships Teacher Guide and Keys

Target preparation and interpretation

The target is reasoning about a whole relationship: a common output/input constant for every allowed input, connected table/rule/graph/point meaning and a claim limited to the supplied data or model. Students move beyond applying a stated Grade 6 constant rate. This selected Grade 7 supplement addresses parts of proportional-relationship recognition and representation; it does not cover fraction-rate fluency, all percent problems or an entire grade/course.

Teach in the supplied order

- Use Starting task before discussion when current reasoning is unknown. Its first problem checks the earlier constant-rate prerequisite; the next problems probe equal-increase and finite-record claims.
- In Test the whole relationship, model both tanks aloud: identify the output, test more than one positive-input ratio, and treat zero as a pair rather than a division. Ask the learner to complete the different practice.
- In Connect table, rule and graph, name the horizontal input and vertical output. Explain why a straight graph alone is not enough. Introduce ordered pairs, the origin and the one-unit point before requesting a drawing.
- In What a model does and does not establish, compare two possibilities that fit one finite record. Teach a conditional prediction and an allowed-input domain. A table may be proportional across its displayed pairs without establishing an entire physical process.
- Give Independent check after teaching. Choose actual response teaching below, then Follow-up check after another opportunity. Later transfer changes the setting and adds square-area reasoning supplied in that task.

What counts as evidence

Look separately at quantity direction/units, multiple positive-input ratios, zero meaning, rule and point connection, and claim limits. A valid starting-point counterexample can refute proportionality even if a requested additional ratio calculation is incomplete; record both facts. An arithmetic slip, unfamiliar directions or an access barrier is not automatically a conceptual misconception. Ask a neutral “What does each number measure?” or “Which pair supports that claim?” before deciding.

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Response teaching access and exposure

Observed response	Actual next teaching and return
One or two pairs are treated as enough despite another shown pair	Test every displayed pair. Then return to a different unused classification task.
Equal increases, a straight line or output at one unit is used despite a nonzero start	Check the starting value. Distinguish amount added from total output before returning to the original target.
Quantity direction, labels or coordinate order is unclear	Name the quantities and points. If per-one meaning is missing, use the unchanged foundation lesson/guide in the ZIP; then return to this target.
The classification is defensible but representations or limits need explanation	Connect your representations; ask what k, (0,0), (1,k) and the domain mean.
One finite table or one matching pair is treated as a universal rule	One pair, two possible rules. Compare alternatives and state what more is needed.

The response files contain their own modeling or clarification and different learner work; they are not empty instructions to invent a reteach. Give the chosen file rather than every route. Secure reasoning can move to consolidation or deeper transfer; do not assign remedial work solely because a drawing is omitted under an agreed alternative response method.

Provide the numerical table and point descriptions as visual alternatives. Read-aloud, enlarged print, oral/drawn/labeled-table explanation or extra processing time may preserve this reasoning target. Calculating ratios, supplying the rule/classification, locating the decisive point or coaching the counterexample changes independence. If graph drawing is itself a local target, an oral description is different evidence; state that local change. Record support.

Keep unused Starting task, Independent check, Follow-up check and Later transfer separate from keys and worked discussion. After exposure or substantive help, treat that task as practice. New amounts do not make equal-difficulty forms, and two questions in one model are not independent confirmations. The local record supports a qualified next decision, not diagnosis, mastery or stable-ability ranking.

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Keys Starting task

1 Stated constant rate

12 ÷ 4 = 3 liters per minute. At that same stated constant rate, 7 minutes gives 3 × 7 = 21 liters gained. Accept equivalent fractions, equal-group tables, diagrams or an oral explanation with direction and units. The condition is that the stated model continues at the same rate through the requested time; this is supplied in the problem.

2 Leilas equalincrease claim

Disagree about total water. At zero minutes there are 5 liters, not zero. Positive-time quotients are 8, 11/2 = 5.5 and 17/4 = 4.25 liters per minute, so no one proportionality constant fits. The rate of increase is 3 liters each minute; a matching rule is total = 3 × time + 5. Starting water distinguishes total from water added. A correct zero-input refutation is sufficient evidence against the claim, even if a learner does not compute every quotient. Do not compute 5 ÷ 0.

3 Finite record

3/1 = 6/2 = 12/4 = 3 liters per minute across the displayed pairs. Exactly 18 liters at 6 minutes is not established because no later rule or starting amount is supplied. Accept “18 liters if this proportional filling rule continues through 6 minutes” with the assumption stated. Different later readings could agree with all three displayed values. Do not mark a conditional prediction wrong merely because it is conditional; do not accept an unconditional claim that the entire future process has been proved.

Interpretation

Correct problem 1 but not 2 suggests checking a relationship/output assumption rather than repeating per-one division. Correct displayed ratios but an unconditional claim in 3 calls for model-limit teaching. Unclear work needs a neutral probe. The starting set is a local entry task, not a calibrated placement test.

Proportional Relationships Teacher Guide and Keys

Keys Test the whole relationship

Supplied model

Tank A has k = 3 liters per minute and total y = 3x on 0 ≤ x ≤ 4; zero gives zero. Tank B has y = 3x + 5. Its positive-time quotients are 8, 5.5 and 4.25. Water added in either is 3x; B total includes 5 liters already present. The table, rule and interpretation must refer to the same output.

Guided cord pieces

12/1 = 36/3 = 60/5 = 12 centimeters per piece. L = 12n for whole-number n from 0 to 5. Zero pieces gives zero cord length. Four pieces gives 48 centimeters. Fractional n is outside this complete-piece model, even though the algebraic expression could be evaluated there. Accept correct equal-group reasoning as well as division.

Practice A

No proportionality of total water to time. The positive-time quotients are 6, 10/3 and 14/5 = 2.8 liters per minute; any two unequal quotients refute a common constant. At time zero total is 4 liters. The supplied total rule is y = 2x + 4 on 0 ≤ x ≤ 5. Its rate of increase is 2 liters per minute, but that is not a proportionality constant for this total output.

Practice B

Quotients are 2, 2 and 7/3. The pair (3,7) contradicts y = 2x, which would give 6. Therefore one k does not fit the whole displayed set. No rule for unseen inputs follows from these values alone. A learner who says “the first two are proportional but the whole table is not” is correct. Do not require a particular physical explanation for the third value.

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Keys Connect table rule and graph

Supplied graphs

Empty-start points are (0,0), (1,2), (2,4), (4,8), with y = 2x and k = 2 liters per minute. The other points are (0,3), (1,5), (2,7), (4,11), with y = 2x + 3. Both are straight over 0 to 4 minutes; the second does not pass through the origin. In the second, output at one minute includes the starting water and is not the rate of increase.

Guided fractional constant

The completed y column is 0, 1.5, 3, 6. k = 1.5 = 3/2 liters per minute; y = 1.5x or y = (3/2)x. Plot/describe (0,0), (1,1.5), (2,3), (4,6), with time horizontal and total water vertical. Join only the stated continuous-time domain 0 to 4. Zero elapsed time gives zero water; one minute gives 1.5 liters. Half a grid step at 1.5 is intentional, not rounding.

Aris practice claim

The claim is false. The zero point is (0,2), and y = 4x predicts 8 liters at 2 minutes while the table gives 6. Positive-time quotients are 4, 3 and 2.5 liters per minute. The correct supplied-model rule is y = 2x + 2. Points are (0,2), (1,4), (2,6), (4,10). A straight graph or one matching point alone is insufficient. An accurate point description can preserve the reasoning goal when drawing is not the locally assessed skill.

Proportional Relationships Teacher Guide and Keys

Keys Model limits consolidation and clarification

What a model does and does not establish

The two given possibilities fit (1,3), (2,6), (4,12), but give 18 and 16 liters at 6 minutes. The finite record alone does not choose a future rule. For cord displays A: L = 2n, proportional with k = 2 meters per piece; B: L = 2n + 4, not proportional to new-piece count. At n = 3 the totals are 6 and 10 meters. A fractional count 3.5 is outside the stated complete-piece domain 0 to 6. A different physical model allowing cut pieces would be a changed condition.

In the unknown practice record, 4/1 = 8/2 = 16/4 = 4 liters per minute for displayed pairs. A 5-minute prediction of 20 liters requires the proportional rule to continue through that time; it is not established. Square-area quotients 1/1 = 1 and 4/2 = 2 differ, so a zero pair alone does not make area proportional to side length. The proposed output and its units matter.

Connect your representations

All positive ratios equal 2.5 = 5/2 liters per minute; zero pair is (0,0). Rule y = 2.5x. (1,2.5) means 2.5 liters after one minute. Three minutes gives 7.5 liters. Six minutes is outside the given 0-to-4-minute domain, so no reading there is guaranteed by the stated model. Accept 15 liters as a prediction only if the rule is explicitly extended as an assumption, not as supplied fact.

Name the quantities and points

(3,4.5) means 4.5 meters of cord after 3 minutes. Positive quotients 1.5/1, 3/2 and 4.5/3 are all 1.5 meters per minute. Rule y = 1.5x for $0 \le x \le 3$; origin gives no delivered cord at zero time; (1,1.5) is the one-minute amount. Its straight graph passes through the origin over that domain. Minutes per meter is 2/3, a different requested direction; do not label it meters per minute.

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Keys Targeted reteaching and deeper reasoning

Test every displayed pair

Model quotients are 5, 5 and 16/3; y = 5x fails at x = 3. Practice quotients are 2, 2 and 10/4 = 2.5. y = 2x gives 8, not 10, at x = 4. No common k fits every displayed pair. An unseen x = 3 output is unknown without a further rule; 6 would require an additional assumption and still would not make the entire shown table proportional.

Check the starting value

Model total y = 2x + 4 has positive quotients 6 and 4 and a nonzero start. Practice total y = 3x + 2 has quotients 5, 4 and 14/4 = 3.5 liters per minute and starts at (0,2), so total is not proportional to time. Water added is a = 3x on the stated 0-to-4 domain, proportional with 3 liters per minute and zero added at time zero. Accept any two unequal positive ratios and a clear distinction between total and added amounts.

One pair two possible rules

At x = 2, both P = 5 × 2 = 10 and Q = 3 × 2 + 4 = 10 liters. P is proportional, with k = 5 liters per minute and origin (0,0); Q starts at 4 and is not proportional. Many valid discriminating inputs exist: at x = 0, P gives 0 and Q gives 4; at x = 1, P gives 5 and Q gives 7; at x = 4, P gives 20 and Q gives 16. Any allowed x other than 2 discriminates these two rules. One reading distinguishes these candidates under their supplied conditions; it does not prove an unrestricted future physical law or establish values outside 0 to 4 minutes.

Proportional Relationships Teacher Guide and Keys

Keys Independent check

1 Totalwater claim

The claim about proportional total is false. Positive-time quotients are 14/2 = 7, 22/4 = 5.5 and 30/6 = 5 liters per minute. Any two unequal quotients refute one proportionality constant. The zero pair (0,6) is another counterexample. The total rule is $y = 4x + 6$ for $0 \le x \le 6$. Four liters per minute is the added-water rate; it is not a common total-water/time quotient. If a correct starting-point refutation lacks the requested ratios, record sound classification plus incomplete ratio evidence rather than calling the classification wrong.

2 Emptystart representation

Completed y values: 0, 1.75, 3.5, 7. k = 1.75 = 7/4 liters per minute; $y = 1.75x$. Correct points: (0,0), (1,1.75), (2,3.5), (4,7). Time is horizontal, water vertical. A straight origin-passing graph is valid over the given continuous-time domain. Zero time gives zero water; the one-minute point gives 1.75 liters. Quarter-grid positions are intentional; keep exact values rather than rounding. Equivalent exact decimal/fraction expressions and accurate oral/labeled-table point descriptions are acceptable for this reasoning target.

3 Unknown record

3/1 = 9/3 = 15/5 = 3 liters per minute for the displayed pairs. Exactly 21 liters at 7 minutes is not established because there is no supplied later rule. Accept a conditional 21-liter prediction if the same proportional rule continues through 7 minutes. “Not enough information for an unconditional exact prediction” is defensible, not an omission.

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Keys Followup check

1 Totalcord claim

The claim is false. Positive-time quotients are 5/1 = 5, 9/3 = 3 and 13/5 = 2.6 meters per minute. Any two unequal quotients refute a common k. At time zero, total cord is 3 meters. The supplied total rule is $y = 2x + 3$ for $0 \le x \le 5$. Two meters per minute is the rate of added cord, not a constant total-cord/time quotient. Distinguish a correct counterexample from completion of every requested representation.

2 Emptystart representation

Completed y values: 0, 2.25, 4.5, 9. k = 2.25 = 9/4 liters per minute; $y = 2.25x$. Points: (0,0), (1,2.25), (2,4.5), (4,9). The straight graph passes through the origin for the stated continuous-time domain. At zero time there is zero water; at one minute there are 2.25 liters. Quarter-grid positions are intentional; keep exact values rather than rounding. Accept exact equivalent representations and the agreed response method.

3 Unknown record

6/1 = 12/2 = 30/5 = 6 meters per minute across the displayed pairs. Exactly 36 meters at 6 minutes requires the same proportional model to continue through 6 minutes. The finite record alone does not establish that future value. No equal-difficulty or mastery inference follows from comparing this check with the preceding one.

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Keys Later transfer and use boundaries

1 Ribbon mass and bag

For lengths 0, 1, 2.5 and 6 meters, A gives masses 0, 3, 7.5 and 18 grams. B gives 4, 7, 11.5 and 22 grams. A: y = 3x, with k = 3 grams per meter and origin (0,0). Its positive-input quotients are all 3. B: y = 3x + 4, with positive-input quotients 7, 11.5/2.5 = 4.6 and 22/6 = 11/3 grams per meter. B begins at (0,4), so the total including bag is not proportional to ribbon length. At one meter, A is 3 grams and B is 7 grams including the bag. At 8 meters, A is 24 grams and B is 28 grams. Ten meters is outside the stated 0-to-8 domain; 30 and 34 grams would require extending the model as a stated assumption.

2 Square area

Positive-side quotients are 1/1 = 1, 4/2 = 2 and 9/3 = 3 square centimeters per centimeter, so area is not proportional to side length. Origin alone is insufficient. A = 3s matches (3,9) but at s = 1 predicts 3, not 1, or at s = 2 predicts 6, not 4. The supplied area rule gives 4 × 4 = 16 square centimeters at side 4 centimeters, within its domain. Accept either displayed counterexample and correct units; an invented constant matching one pair is not proof of a whole relationship.

Sources and classroom use

Tasks, graphs, fictional learner responses and numerical data are original Mentor Teaching instructional constructions. Selected Grade 7 fit references recognition/representation in CCSS 7.RP.A.2, including table tests, constants, equations and origin/one-unit point meaning: https://www.thecorestandards.org/Math/Content/7/RP/. This is a scope reference, not complete-standard alignment or classroom validation. Teachers may print and adapt these materials under the site Terms of Use, keeping attribution and identifying changes. Word pagination may change after editing; check print preview.

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Fictional teaching response — sound for the named task

Named task: Independent check 1: tank starts with 6 liters and gains 4 liters per minute

Fictional response — sound for the named task: “At 2 minutes, 14 ÷ 2 = 7 L/min; at 4 minutes, 22 ÷ 4 = 5.5 L/min. Those total/time quotients differ, and the zero-input pair is (0, 6), not (0, 0). Total water follows y = 4x + 6. Four L/min is the change rate, not a constant of proportionality for total water.”

Visible evidence: Uses two positive-time quotients, the zero-input pair and the supplied rule to distinguish a linear increase from proportional total water.

Not established: Physical validation of a tank or a general rule from unmodeled measurements; the scenario supplies the model.

Clarify: Ask which quantity y represents: total water or only water added.

Next teaching/evidence: Use Follow up check after another learning opportunity, and keep Later transfer for later.

This response is invented for teacher interpretation. Accept defensible alternatives. Record access, prior exposure and the actual response; do not assign an ability label or claim mastery from this example.

Follow up check

Reserve: Later transfer

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Fictional teaching response — partially developed

Named task: Independent check 1: tank starts with 6 liters and gains 4 liters per minute

Fictional response — partially developed: “It is not proportional. The rule is y = 4x + 6.”

Visible evidence: The correct conclusion and rule.

Not established: The requested quotient comparisons, zero-input interpretation or distinction between change rate and total/time ratio.

Clarify: Ask for the two displayed quotient calculations and what the 6 means. An absent explanation does not prove the learner lacks it.

Next teaching/evidence: If the starting amount is overlooked in the explanation, use Check the starting value before the separate follow-up check.

This response is invented for teacher interpretation. Accept defensible alternatives. Record access, prior exposure and the actual response; do not assign an ability label or claim mastery from this example.

Check the starting value

Reserve: Later transfer

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Fictional teaching response — plausible error

Named task: Independent check 1: tank starts with 6 liters and gains 4 liters per minute

Fictional response — plausible error: “It gains 8 liters every 2 minutes, so y = 4x and k = 4 for total water.”

Visible evidence: Correct change rate over the equal intervals.

Not established: Proportional total water: the proposed rule predicts zero at time zero, contrary to the supplied 6-liter start.

Clarify: Substitute x = 0 in that rule, compare the table and distinguish added water from total water.

Next teaching/evidence: Use Check the starting value, including its new 2-liter-start, 3-L/min practice. Compare total and added-water rules there. Reserve Follow up check and Later transfer.

This response is invented for teacher interpretation. Accept defensible alternatives. Record access, prior exposure and the actual response; do not assign an ability label or claim mastery from this example.

Check the starting value

Reserve: Later transfer