

Find Rate from Changes

Use both changes, not the whole output

For a linear relationship, change per input unit is $m = \text{change in output} \div \text{change in input}$. Use two different inputs, and subtract in the same order: $m = (\text{later output} - \text{earlier output}) \div (\text{later input} - \text{earlier input})$. Do not divide by a zero input change.

Model a container with water already present

A supplied linear model allows time from 0 to 5 minutes. Time is input; total water is output.

Time (minutes)	Total water (liters)
0	5
2	9
5	15

From 0 to 2 minutes: $(9 - 5) \div (2 - 0) = 4 \div 2 = 2$ liters per minute. From 2 to 5: $(15 - 9) \div (5 - 2) = 6 \div 3 = 2$ liters per minute. The 4-liter and 6-liter jumps differ because the time intervals differ. Both give the same per-minute change.

At two minutes, $9 \div 2 = 4.5$ liters per minute is total water divided by elapsed time. It includes the starting water and is not this model's change rate. With an empty start, the proportionality constant and change rate can be the same.

Explain which quantities belong in the rate calculation

Find Rate from Changes

Practice unequal intervals

1 Guided practice

A supplied linear model gives the following total cord lengths for 0 to 7 minutes. Calculate the rate over each interval. Explain why equal rates need not give equal output jumps here.

Time (minutes)	Total cord (meters)
0	4
3	13
7	25

Both interval rates and units

2 Your practice

A supplied linear total-water model has points (1, 8.5), (3, 11.5) and (6, 16), with minutes first and liters second. Find the rate between the first two and last two points. Then test the claim “ $8.5 \div 1$ is the change per minute.”

Your interval calculations and response

Find Rate from Changes

A decrease or no change is still a rate

A change from 12 liters to 8 liters is $8 - 12 = -4$ liters. Over two minutes, the rate is $-4 \div 2 = -2$ liters per minute. The negative sign means the amount remaining decreases; it does not mean the starting amount was negative.

3 Decreasing amount

A supplied linear model of amount remaining, valid from 0 to 4 minutes, has points (1, 14) and (3, 8). Find the signed change rate and explain its meaning. What output change occurs over one more minute under that model?

Signed change, rate and meaning

4 Constant amount

A supplied constant model has 9 liters at every time from 0 to 6 minutes. Use the points (1, 9) and (4, 9) to find its rate. Explain why a zero rate does not mean zero water.

Your zero-rate explanation
